**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

**Answer :**

We have a normal distribution with µ= 45 and *σ* = 8.0.

Let X be the amount of time it takes to complete the repair on a customer's car. To finish in one hour you must have X ≤ 50 so the question is to find Pr(X > 50).

Pr(X > 50) = 1 - Pr(X ≤ 50).

Z = (X - µ)/ = (X - 45)/8.0

Thus the question can be answered by using the normal table to find

Pr(X ≤ 50) = Pr(Z ≤ (50 - 45)/8.0) = Pr(Z ≤ 0.625)=73.4%

Probability that the service manager will not meet his demand will be = 100-73.4 = 0.2676 or 26.6%

**Corresponds to option B**

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

**Answer :**

**True.** Since the distribution is normal and the mean is 38, which is greater than 44, more employees are older than 44 than between 38 and 44. This is because the normal distribution is symmetrical around the mean.

Probability of employees being older than 44: 0.15865525393145707 or 16%

Probability of employees being between 38 and 44: 0.3413447460685429 or 34%

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**Answer :**

**True.** we find that the expected number of employees under the age of 30 is approximately 35.87. Therefore, the statement "A training program for employees under the age of 30 at the center would be expected to attract about 36 employees" is true.

Expected number of employees under the age of 30: 36.484487890347154

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**Answer :**

The difference between 2X1 and X1 + X2 is that they have the same mean but

Different variances.

2X1:

Distribution: 2X1 follows a normal distribution with mean 2μ and variance 4σ^2.

Parameters: Mean = 2μ, Variance = 4σ^2

X1 + X2:

Distribution: X1 + X2 follows a normal distribution with mean 2μ and variance 2σ^2.

Parameters: Mean = 2μ, Variance = 2σ^2

Both random variables have the same mean (2μ) because they are based on the

same mean (μ). However, their variances differ, with 2X1 having a variance of 4σ^2 and X1 + X2 having a variance of 2σ^2.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

**Answer :**

Since we need to find out the values of a and b, which are symmetric about the mean, such that the probability of random variable taking a value between them is 0.99, we have to work out in reverse order.

The Probability of getting value between a and b should be 0.99.

So the Probability of going wrong, or the Probability outside the a and b area is 0.01 (ie. 1-0.99).

The Probability towards left from a = -0.005 (ie. 0.01/2).

The Probability towards right from b = +0.005 (ie. 0.01/2).

So since we have the probabilities of a and b, we need to calculate X, the random variable at a and b which has got these probabilities.

By finding the Standard Normal Variable Z (Z Value), we can calculate the X values.

Z= (X- μ) / σ

For Probability 0.005 the Z Value is -2.57 (from Z Table).

Z \* σ + μ = X

Z(-0.005)\*20+100 = -(-2.57)\*20+100 = 151.4

Z(+0.005)\*20+100 = (-2.57)\*20+100 = 48.6

So, option D is correct**.**

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45

**Profit1+profit2~N(5+7,3 2+42)= Profit~N(12,5)**

1. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

**Answer :**

Range = 12-19.6×5,12+ 1.96 ×5

=$2.2, $22.8

=Rs.99, Rs.1026

1. Specify the 5th percentile of profit (in Rupees) for the company

**Answer :**

P(Z<=(p-12)/5)=0.05

p-12/5= -1.644

p=12 – 8.22 = $3. 78 = rs.170.1

1. Which of the two divisions has a larger probability of making a loss in a given year?

**Answer :**

When profit is less than 0 then loss

p-12/5= -1.644

p= 12-8.22 - $3.78 = Rs.170.1